ÉRETTSÉGI VIZSGA • 2010. május 4.

# MATEMATIKA ANGOL NYELVEN

## KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

## JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM

# Important Information

**Formal requirements:**

1. The papers must be assessed in **pen and of different colour** than the one used by the candidates. Errors and flaws should be indicated according to ordinary teaching practice.
2. The first one among the shaded rectangles next to each question contains the maximal score for that question. The **score** given by the examiner should be entered into the other **rectangle**.
3. **In case of correct solutions**, it is enough to enter the maximal score into the corresponding rectangle.
4. In case of faulty or incomplete solutions, please indicate the corresponding partial scores within the body of the paper.
5. Nothing, apart from the diagrams, can be evaluated if written in pencil.

**Substantial requirements:**

1. In case of some problems there are more than one marking schemes given. However, if you happen to come across with some **solution different** from those outlined here, please identify the parts equivalent to those in the solution provided here and do your marking accordingly.
2. The scores in this assessment **can be split further**. Keep in mind, however, that the number of points awarded for any item can be an integer number only.
3. In case of a correct answer and a valid argument the maximal score can be awarded even if the actual solution is **less detailed** than that in this booklet.
4. If there is a **calculation error** or any other flaw in the solution, then the score should be deducted for the actual item only where the error has occured. If the candidate is going on working with the faulty intermediate result and the problem has not suffered substantial damage due to the error, then the subsequent partial scores should be awarded.
5. If there is a **fatal error** within an item (these are separated by double lines in this booklet), then even formally correct steps should not be given any points, whatsoever. However, if the wrong result obtained by the invalid argument is used correctly throughout the subsequent steps, the candidate should be given the maximal score for the remaing parts, unless the problem has been changed essentially due to the error.
6. If an **additional remark** or a **measuring unit** occurs in brackets in this booklet, the solution is complete even if the candidate does not mention it.
7. If there are more than one correct attempts to solve a problem, it is the **one indicated by the candidate that can be marked**.
8. You should **not give any bonus points** (points beyond the maximal score for a solution or for some part of the solution).
9. You should not reduce the score for erroneous calculations or steps unless its results are actually used by the candidate in the course of the solution.
10. **There are only 2 questions to be marked out of the 3 in part II/B of this exam paper.** Hopefully, the candidate has entered the number of the question not to be marked in the square provided for this. Accordingly, this question should not be assessed even if there is some kind of solution contained in the paper. Should there be any ambiguity about the student's request with respect to the question not to be considered, it is the last one in this problem set, by default, that should not be marked.

# I.

| **1.** | | |
|---|---|---|
| The third side is 8 cm long. | 2 points | |
| **Total:** | **2 points** | |

| **2.** | | |
|---|---|---|
| By 8 600. | 2 points | *The score should be reduced by 1 point if the sign is wrong.* |
| **Total:** | **2 points** | |

| **3.** | | |
|---|---|---|
| The coordinates of the vector **a**+**b** are (1; 5). | 2 points | |
| **Total:** | **2 points** | |

| **4.** | | |
|---|---|---|
| $x = -2$. | 2 points | |
| **Total:** | **2 points** | |

| **5.** | | |
|---|---|---|
| *B* and | 1 point | *If there are wrong letters occuring on the list then 0 point must be given.* |
| *C* | 1 point | |
| **Total:** | **2 points** | |

| **6.** | | |
|---|---|---|
| Being familiar with the concept of zero of a function (e.g. $5x-3=0$). | 1 point | |
| For the actual solution: $x=\frac{3}{5}$. | 1 point | |
| **Total:** | **2 points** | |

| **7.** | | |
|---|---|---|
| The brick is 8 cm high. | 2 points | |
| **Total:** | **2 points** | |

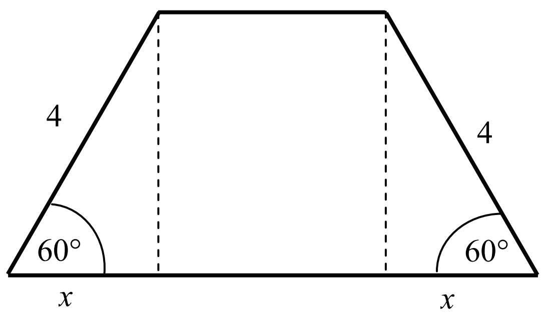
| **8.** | | |
|---|---|---|
| $\frac{47.3}{9460}$; | 2 points | *For the proper ratio.* |
| 47.3 billion km = 0,005 ($=5\cdot10^{-3}$) light years. | 1 point | |
| **Total:** | **3 points** | |

| **9.** | | |
|---|---|---|
| The centre of the circle is (0; –1), | 2 points | *1-1 point for each coordinate.* |
| and its radius is 2 (units) long. | 1 point | |
| **Total:** | **3 points** | |

| **10.** | | |
|---|---|---|
| Some solutions are (1; 2; 6) or (2; 2; 5). | 3 points | *Any correct answer is worth 3 points. If there are some conditions not satisfied than at most 1 point may be given.* |
| **Total:** | **3 points** | |

| **11.** | | |
|---|---|---|
| (The losing candidate received 1632 votes; this number represents the favourable outcomes. The total number of outcomes is the number of people eligible to vote. The probability is $\frac{1632}{12608}(\approx 0.129)$. | 3 points | |
| **Total:** | **3 points** | |

**12.**




| | | |
|---|---|---|
| Correct diagram. | 1 point | |
| $x$ = 2 (cm), either by completing the equilateral triangle or by trigonometry. | 2 points | |
| Hence the shorter base is 7-2$x$ = 3 (cm) long. | 1 point | |
| **Total:** | **4 points** | |

# II/A.

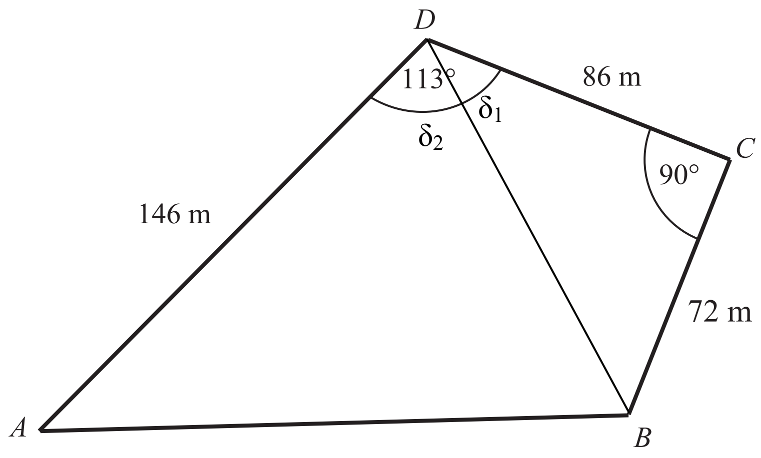
| **13. a)** | | |
|---|---|---|
| The zeros of *f* are $x_1 = 2$, | 1 point | |
| $x_2 = -6$. | 1 point | |
| The range is $[-4; 4]$. | 1 point | *This point should be given for any form of the correct answer.* |
| The smallest value is –4 | 1 point | |
| and it is assumed if $x = -2$. | 1 point | |
| **Total:** | **5 points** | |

| **13. b)** | | |
|---|---|---|
| The rule of the mapping is $f(x) = \lvert x+2 \rvert - 4$. | 4 points | *Each correct constant is worth 2-2 points.* |
| **Total:** | **4 points** | |

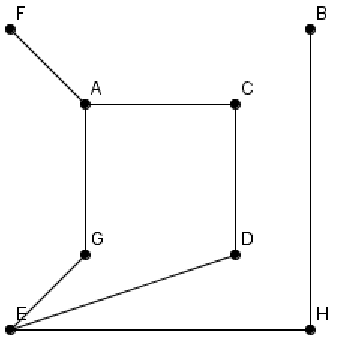
| **13. c)** | | |
|---|---|---|
| $x_1 = 0$ $x_2 = -4$ | 3 points | |
| **Total:** | **3 points** | |

*In case of algebraic solution 1 point is due for the proper separation of the cases and 1-1 for the roots, respectively.*
*In case of graphic solution 1-1 point should be given for the two roots, respectively, and 1 for checking them.*

## 14.

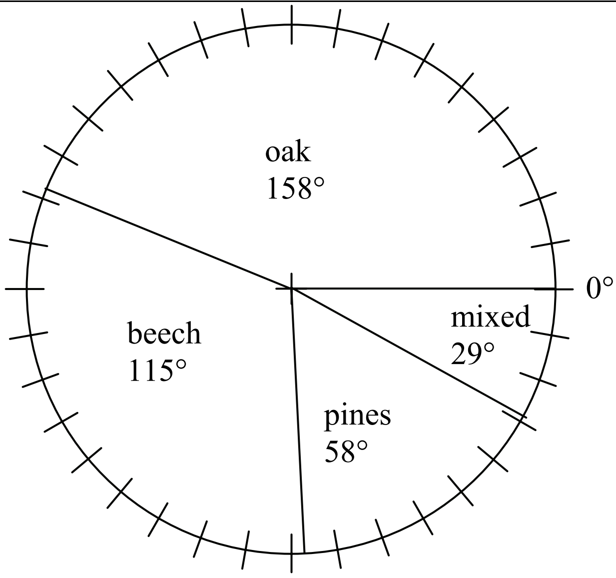



| | | |
|---|---|---|
| Divide the quadrilateral into two triangles. | 1 point | *This point should be given even if this idea is only implicitly present in the solution.* |
| (By Pithagoras' theorem in the triangle *BCD*) $BD^2 = BC^2 + CD^2$. | 1 point | |
| $BD = \sqrt{86^2 + 72^2} = \sqrt{12580} = 112.16$. | 2 points | |
| $\tan\delta_1 = \frac{72}{86}$, | 1 point | |
| $\delta_1 \approx 39.94°$. | 1 point | * $\delta_1 \approx 40°$ *may be accepted* |
| $\delta_2 \approx 113° - 39.94° = 73.06°$. | 1 point | * $\delta_2 \approx 73°$ *may be accepted* |
| $A_{ABD} = \frac{146 \cdot 112.16 \cdot \sin 73.06°}{2} \approx 7832.42$ | 2 points | |
| $A_{BCD} = \frac{86 \cdot 72}{2} = 3096$ | 1 point | |
| (The area of the quadrilateral can be computed as the sum of the respective areas of the two triangles) $A = A_{ABD} + A_{BCD} \approx 7832.42 + 3096 = 10928.42$. | 1 point | |
| The area of the piece of land is 10 900 $m^2$, to the nearest hundred square meters. | 1 point | |
| **Total:** | **12 points** | |

| | | |
|---|---|---|
| **15. a)** | | |
| Since there were seven people to be notified, there must have been at least seven phone calls made. | 2 points | *These 2 points are due even without explanation.* |
| **Total:** | **2 points** | |
| **15. b)** | | |
|   | 6 points | Edges (*Andy – Cecily*), (*Andy– Frank*) -- *1 point.* Edge (*Cecily– Daniel*) -- *1 point.* Edge (*Gill– Andy*) *1 point.* Edge ( *Helen– Bill*) *1 point.* *Ed's three edges are worth 2 points.* *1-1 point should be taken away for each wrong edge.* |
| **Total:** | **6 points** | *A correct diagram with no explanation is still worth 6 points.* |
| **15. c) first solution** | | |
| Out of eight there are $\binom{8}{3} = (56)$ ways to choose three passangers in the first compartment; | 1 point | |
| out of the remaining 5 there are $\binom{5}{3} = (10)$ ways to choose three in the second compartment, | 1 point | |
| finally, the last two go to the third compartment (1 posibility). | 1 point | |
| Therefore, there are 560 ways for the eight friends to be arranged in the three compartments altogether and thus the answer is YES. | 1 point | |
| **Total:** | **4 points** | |
| **15. c) second solution** | | |
| Arrange the eight friends in a row and assign one of the labels C1, C2 or C3 to each of them depending on which compartment will this student be seated. | 1 point | |
| There are three copies of C1, three copies of C2 and, finally, two copies of C3 to be assigned. If these labels were all different then there would be 8! ways of arranging them in a row. | 1 point | |
| This time, however, the 3-3-2 repetitions inside the compartments, respectively, yield $\frac{8!}{3!\cdot 3!\cdot 2!} =$ | 1 point | |
| = 560 arrangements, altogether. | 1 point | |
| **Total:** | **4 points** | |

# II/A.

| 16. a) | | |
|---|---|---|
| The first term of the geometric progression is 29 000 and its common ratio is 1.02. | 1 point | *This 1 point should be given even if this idea is only implicitely present in the solution.* |
| (The amount of wood after 11 years is the 12th term of the progression.) $(a_{12} =)\ a_1 \cdot q^{11} = 29\,000 \cdot 1.02^{11}$. | 2 points | *The correct model is worth 1 point, as well as the actual substitution.* |
| $a_{12} \approx 36\,057.86$. | 1 point | |
| There will be 36 000m$^3$ , to the nearest thousand, wood in the forest after 11 years. | 1 point | |
| **Total:** | **5 points** | |

| 16. b) | | |
|---|---|---|
|  | | |
| As the consecutive terms of a G.P. the amount of trees different from oak (beech, pines and mixed) add up to 56% of the woodstock. | 1 point | *Beeches are b%,the mixed are e% and pines are 16% of the stock.* |
| Pines comprise 16%, beeches comprise 16$q$ % and the mixed trees comprise of $\frac{16}{q}$ % of the stock. | 1 point | $b+e=40$. |
| $\frac{16}{q}+16+16q=56$. <br> $\left(16q^2-40q+16=0\right)$ | 2 points | $\frac{b}{16}=\frac{16}{e}$. *Substitution yields the equation* $e^2-40e+256=0$. |
| Therefore, $q=2$ or $q=0.5$ | 2 points | *Its solution is* $e=32$ *or* $e=8$. |
| Since there are less mixed trees than pines, only $q=2$ is consistent. | 1 point | *Since there are more pines than mixed,* $e=8$ *and* $b=32$. |
| Hence the beeches are 32% and the mixed trees are 8% of the total. | 1 point | |
| Accordingly, the corresponding central angles on the pie chart are as follows: oak: 158°, beech: 115°, pine: 58°, mixed: 29°, respectively. | 2 points | *If there is one wrong angle then 1 point may be given; 0 points if there are more.* |
| Correct pie chart. | 2 points | |
| **Total:** | **12 points** | |

| **17. a)** | | |
|---|---|---|
| With the exception of 0°; 90°; 180°; 270°; 360° any angle in the given range can be substituted. | 2 points | *If there are wrong angles occurring on the list (e.g. not in the given interval) or some values (1 or 2) are missing, then at most 1 point may be given.* |
| $\frac{4}{\tan x} = 5 - \tan x$. | 1 point | |
| $\tan^2 x - 5\tan x + 4 = 0$. | 1 point | |
| Hence $\tan x = 1$ or | 1 point | |
| $\tan x = 4$. | 1 point | |
| Hence $x_1 = 45°$; $x_2 = 225°$ | 2 points | |
| $x_3 \approx 75.96°$; $x_4 \approx 255.96°$. | 2 points | |
| Each of the four values are solutions. | 1 point | *Numerical checking may be accepted.* |
| **Total:** | **11 points** | |

| **17. b)** | | |
|---|---|---|
| $\lg(x-3) + \lg 10 = \lg x$. | 2 points | |
| $\lg 10(x-3) = \lg x$. | 1 point | |
| (Either by the monotonicity or by the bijectivity of the logarithmic function) $10(x-3) = x$. | 1 point | |
| $x = \frac{10}{3}$. | 1 point | |
| Checking. | 1 point | |
| **Total:** | **6 points** | |

| **18. a) first solution** | | |
|---|---|---|
| In this selection there are 88 ways of choosing a good item in each of the 6 trials, therefore there are $88^6$ favourable outcomes. | 2 points | *If this idea appears only as the correct formula thenat most 2 points may be given.* |
| There are 100 ways of choosing in each of the six draws, therefore the total number of outcomes is $100^6$. | 2 points | |
| The probability is hence $\frac{88^6}{100^6} = 0.88^6 \approx 0.4644$. | 1 point | *Accuracy to two decimal places may be accepted.* |
| **Total:** | **5 points** | |
| **18. a) second solution** | | |
| The probability of choosing a good item is 0.88. | 1 point | *If this idea appears only as the correct formula then only 2 points may be given.* |
| The subsequent selections are independent | 1 point | |
| and there are six repetitions. | 2 points | |
| The probability is hence $0.88^6 \approx 0.4644$. | 1 point | |
| **Total:** | **5 points** | |

| **18. b)** | | |
|---|---|---|
| *First event („no defected ones in the sample”).* | | |
| There are $\binom{100}{6}$ ways to select 6 out of 100; this is the total number of outcomes. | 2 points | |
| There are $\binom{88}{6}$ ways to choose 6 good items, therefore | 1 point | |
| the probability of the first event is $\frac{\binom{88}{6}}{\binom{100}{6}}$. | 1 point | |
| This probability is equal to $\frac{88\cdot 87\cdot 86\cdot 85\cdot 84\cdot 83}{100\cdot 99\cdot 98\cdot 97\cdot 96\cdot 95}=\frac{541\,931\,236}{1\,192\,052\,400}\approx 0.455$. | 1 point* | *Accuracy to two decimal places may be accepted.* |
| *Second event : („there are at least two defected items in the sample”).* **First solution.** | | |
| In the complemantary event „*there is at most 1 faulty item in the sample*”. | 2 points | |
| This latter event is the disjunctive sum of two mutually exclusive events, namely „*there is no faulty item*” and „*there is exactly one faulty item* ”. | 1 point | *These points are due even if the argument becomes apparent from the actual solution.* |
| Adding the probabilities of these two events one gets the probability of the complementary event, as: | 1 point | |
| $\frac{\binom{88}{6}}{\binom{100}{6}}+\frac{\binom{88}{5}\cdot\binom{12}{1}}{\binom{100}{6}}\approx 0.455+0.394=0.849$ | 1 point | *Accuracy to two decimal places may be accepted. If the candidate has been working according to to this level of accuracy then 0.84 may also be accepted.* |
| Therefore, the probability of having at least two faulty items is 1–0,849, i.e. cca. 0,151. | 1 point* | *Accuracy to two decimal places may be accepted, and working so the candidate might have received 0.16 which then may be accepted.* |

| *Second event („there are at least two faulty items”).* | **Second solution** | |
|---|---|---|
| One simply adds the respective probabilities of the events a *„there are exactly 2, 3 4, 5, 6 faulty items”* $P(X=2)=\frac{\binom{88}{4}\cdot\binom{12}{2}}{\binom{100}{6}}\left(\approx 0.1291\right)$ $P(X=3)=\frac{\binom{88}{3}\cdot\binom{12}{3}}{\binom{100}{6}}\left(\approx 0.0203\right)$ $P(X=4)=\frac{\binom{88}{2}\cdot\binom{12}{4}}{\binom{100}{6}}\left(\approx 0.0016\right)$ $P(X=5)=\frac{\binom{88}{1}\cdot\binom{12}{5}}{\binom{100}{6}}\left(\approx 5.85\cdot 10^{-5}\right)$ $P(X=6)=\frac{\binom{88}{0}\cdot\binom{12}{6}}{\binom{100}{6}}\left(\approx 7.7\cdot 10^{-7}\right)$. | 5 points | *1–1 point for each event.* |
| The probability of „*there are at least two faulty items*” is: ≈0.1291 + 0.0203 + 0.0016 + 0.0001 + 0.000 = = 0.151, | 1 point* | *Accuracy to two decimal places may be accepted.* |
| | | |
| | | |
| therefore, the first event is more probable. | 1 point* | |
| **Total:** | **12 points** | |
| *The 3 points marked by * may be also given if, instead of calculating their numerical value, the candidate compares the given probabilities in some other way.* | | |